Structural change in manufacturing and services: A 2D Model

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Abstract In this paper , I attempt to form a 2 dimensional model of wage share and employment. However, unlike the classical Goodwinian model (which is based on the Lotka-Voltera system), where wage share and employment rate are variables for a specific economy, here I am considering the effect of changes in employment in the services sector on the wage share of manufacturing sector. The intuitive idea is that an increase in the employment of labor in service sector weakens the labor share of the manufacturing sector workers. The underlying reason is the relative lower bargaining power of workers in service sectors (lower levels of unionization, tougher price competition and attempts to reduce costs through globalization could be some of the reasons behind this). This lower bargaining power reduces the manufacturing labor share (and might also reduce the aggregate labor share). I show that with given model specifications, there can be a stable low employment share, low labor share equilibrium. But as the feedback from service sector employment to manufacturing sector wages becomes weaker, the fixed points become unstable and the relationship between these variables cannot be predicted. Of course many simplistic assumptions are required to satisfy the conclusions of the model and the model generates a stable spiral for specific values of parameters and very close to the fixed point.

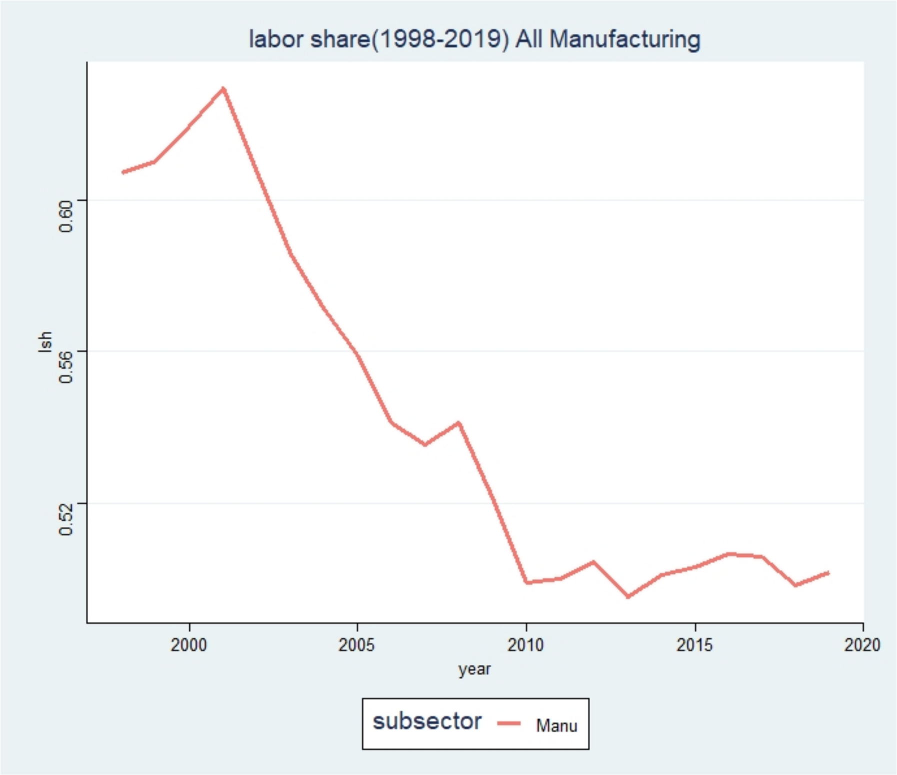
Contents

I. Introduction

In February 2022, Starbucks' workers in about 60 stores across 19 states announced public workers union drive. The era of the weaker bargaining position of laborers in the services sector seems to be coming to an end. This movement is not just restricted to Starbucks or Amazon or Uber, but to a whole range of service sector workers. On the other hand, US still has a large manufacturing sector which has also been in a stable decline in employment and labor share (Figure 1). Manufacturing sector has also seen a decline in unionization over time. The US manufacturing sector added about 2370.9 bn USD in the year 2019 to the US economy and employed close to 12.5 million laborers. The average annual growth rate of value-added from 1997-2019 was 2.44 and the growth rate of the nominal wage bill was 1.52, which implies that the share of labor in value added declined. On the other hand, employment in the services sector has been increasing, leading to the idea that structural change in US has been biased towards services, while relying more and more on imported labor and raw materials to sustain its manufacturing base. The increasing concentration of services employment can weaken the bargaining power of laborers in other industries. In labor markets, quite often manufacturing and services sectors are seen as standalone sectors with differences in labor productivity, wage rate and employment levels. In this paper, I try to see if there is a possibility of both these sectors to affect each other. Is there a steady state equilibrium between employment share in services sector and the labor share in manufacturing and to what extent is this stability sensitive to changes in initial conditions and parameter values.

The importance of manufacturing sector in developed and developing economies was a major theme in economics during the second and third phase of industrialization(1910-1990s). Work by many scholars (Rosenstein-Rodan 1961 ,Chenery 1960) stress the

FIGURE 1. *Decline in labor share in manufacturing - Compiled from Beaureau of Econ Analysis*



importance of the manufacturing sector as the one with strong forward and backward linkages. In simple terms, strong linkages would mean that an increase in demand for a manufactured good (such as the automobile sector) generates employment for downstream as well as upstream sectors. Certain manufacturing sub sectors such as intermediate goods, transport equipment and machinery (which are not consumer durable goods) also experience economies of scale and hence increasing returns to scale. With the decline of manufacturing labor share there has also been a subsequent increase in services sector employment in the US as a share of total employment. Thus a theory should be able to explain a relationship between these two variables. Some of the recent literature on labor share decline and structural change have relied on the theories based on the Lewis dynamics and Baumol dynamics (Rada, Schiavone, Arnim, et al. 2021) or focused on technical change where productivity growth is endogenized (Tavani and Zamparelli 2017). In the classical Goodwin framework(Goodwin 1982 and Desai et al. 2006), each of the state variables is for a particular sector or economy, thus the effect of increasing employment in one sector on the wage share of another sector is a critical area to see the effects of structural change (via employment changes) on the labor share. Some of the work on Goodwinian systems (Rada, Schiavone, Arnim, et al. 2021),Flaschel and Greiner 2009) have proposed the extensions of a simple 2D model to capture changes in the economy. Ofcourse, the changes to employment and labor share that are observed in the US economy cannot be simply explained by a few variables alone and higher order systems have also been proposed (for a 3d system with nonlinear technology Tavani and Zamparelli 2017 and Lima 2004).

I first specify the Baseline Model and find the stability properties of the 2D system.There are two variables of interest : labor share and employment share. The labor share of a sector is a ratio of the total compensation made to workers divided by the total value added or output of the sector. In other words, it is the part of the total economic pie which accrues to workers. Decline in labor share is one of the components of rising inequality in the US. For this analysis I am looking at the labor share of the manufacturing sector. The employment share is the employment of the services sector divided by the total employment in the US. The system has a stable spiral for certain values of the parameters. Based on the phase plane analysis, I try to explain the trajectory of labor share and the possibility of instability in a simple 2D model.

II. Baseline Model

The following variables are defined for the 2D model.

ω_m refers to the labor share in the manufacturing sector L_s refers to the employment share in services sector

The dynamical system consisting of these two variables defined above is given by :

$$\dot{\omega_m} = \alpha_m \omega_m (1 + \beta_m \omega_m) + \gamma_m L_s \tag{1}$$

$$\dot{L_s} = \alpha_s L_s + \gamma_s \omega_m - a \tag{2}$$

, where dots denote the time derivative of the variables. According to equation (1), the rate of change of labor share in the manufacturing sector initially rises with a a higher labor share(ω) (this is assumed with $\alpha_m > 0$). However, beyond a certain limit, the increasing costs of labor to firms leads to a decline in the rate of growth of labor share (this could come about with cuts in nominal wages, increases in prices or reducing employment in the manufacturing sector).In order to show this effect we would have to define ($\beta_m < 0$). Apart from this quadratic term, $\dot{\omega_m}$ also depends on the employment in services industry. This characterization of wage share dynamics as a nonlinear function of wage share is not usually done in many models (see Dutt 1984 for how markups can be non-linearly related to its own growth rate or Lima 2004 for an elegant representation of post-Keynesian model with non-linear technological change). However to make the model more realistic, I believe that labor share growth has to be modelled in a non-linear fasion. The other important parameter $\gamma_m < 0$ implies that: higher employment in the services industry reduces the bargaining power for laborers across all industries (including manufacturing) since service sector employment is often characterized by thicker labor markets where employment can be easily based on hire and fire policies. These sectors are also often characterized by no social safety nets, insurance. They are also less unionized and also receive a lower pay than manufacturing sector workers. Increasing servicification of the economy, comes at the cost of reducing the labor share growth of manufacturing.

In equation (2), the growth rate of labor force in the services sector depends positively on the level of employment in the services sector and positively on the labor share of the manufacturing sector. The idea is that an increase in labor share for manufacturing workers, increases the possibilities of bargaining for other labor markets. This might lead to an increases in employment in other sectors. (this requires us to have $\gamma_s > 0$ and $\alpha_s > 0$). The rate of growth of employment in services $\dot{L_s}$ declines with an increasing labor share in manufacturing. The term"a" captures the natural limits to growth rate of labor supply in the services sector (this could be the growth of output in services sector or the growth rate of population). In neo-Goodwin models where employment rate is a negative function of labor productivity growth rate (Rada, Schiavone, Arnim, et al. 2021). "a" is a positive constant and can be interpreted as the exogenous labor productivity growth of the services sector. Since the services sector is also a low productivity sector, small values of "a" are feasible.

Based on the above analysis, I give the following signs to the coefficients in each equation:

$\alpha_m > 0, \beta_m < 0, \gamma_m < 0, \alpha_s > 0, \gamma_s > 0$

III.Stability analysis

The system given by equations 1 and 2 can be used to solve for the steady state dynamics. The steady state solutions for this system can be obtained by setting $\dot{L_s} = 0$, which yields :

$$(L_s)^* = -\frac{a - \gamma_s \omega_m^*}{\alpha_s} \tag{3}$$

The employment level (or as a ratio of total employment in the services sector) cannot be negative. SO $a > \gamma_s \omega_m$. Similarly, by setting $\dot{\omega_m} = 0$ and using the value of $(L_s)^*$ from the above equation, we get the following quadratic equation for ω :

$$(\alpha_m \alpha_s \beta_m)\omega_m^2 + (\alpha_m \alpha_s - \gamma_s \gamma_m)\omega_m + a * \gamma_m = 0 \tag{4}$$

This quadratic equation will solve for ω_m^* and finally, putting the value of ω_m from 4 in 3, we find the steady state value of L_s^*.

In order to analyze the stability properties of the system and to keep the system tractable. I numerically simulate the system for different values of the parameters. I focus on the γ_m parameter which links the labor share in manufacturing to the employment in services. The nullclines and the Jacobian for the system given by equations 1 and 2 is shown below.

$$\begin{bmatrix} \alpha_m + 2\alpha_m \beta_m \omega_m & \gamma_m \\ \gamma_s & \alpha_s \end{bmatrix}$$

The nullclines for the system are shown in Figure 2. With ω_m as the x-axis variable and L_s as the y axis variable. After some further calculations, the conditions for stability of the system : Det(J*) > 0 requires $\alpha_m * \alpha_s > (\gamma_m \gamma_s)/(1 + 2 * \beta_m \omega_m)2$, which is ensured since $\alpha_m \alpha_s$ is always positive based on our assumptions(note that $\gamma_m < 0$. Trace (J*) < 0 requires $\alpha_m < \alpha_s + 2\gamma_s \gamma_m/\alpha_s$. The conditions for stability of the system require some strong assumptions. The β_m term must be large negative to ensure that labor share has a self correcting mechanism (because of the non-linear term in the equation) and also a propelling linear term. To see the clearer picture, I calibrate the model with the same values of all the parameters except the value of γ_m

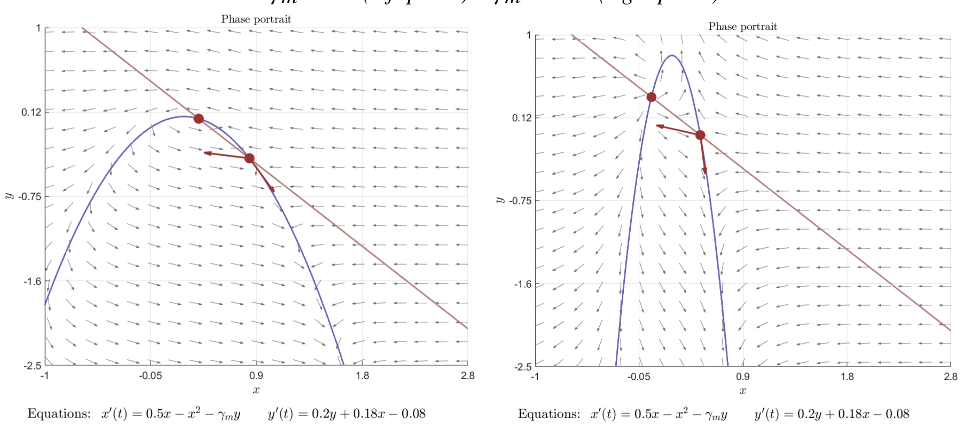
which is what determines the stability. Figure 1 and 2 shows the nullclines and the fixed points in red on the graph. Stability behavior changes across the two systems. The fitted model is given by:

$$\dot{\omega_m} = 0.5\omega_m - \omega_m^2 - \gamma_m L_s \quad (5)$$

$$\dot{L_s} = 0.2 * L_s + 0.18 * \omega_m - 0.08 \quad (6)$$

Two possibilities that might occur with differing values of γ_m are observed. Specifically, I show that as the dependence of manufacturing sector's labor share growth on service sector employment decreases, the steady state stability of the system vanishes. The left panel shows 5 when $\gamma_m = 0.8$ and the right panel when $\gamma_m = 0.08$.

FIGURE 2. $\gamma_m = 0.8$ *(left panel) :* $\gamma_m = 0.08$ *(right panel)*



From the two figures, its observed that the in the right panel when $\gamma_m = 0.08$, we have an unstable spiral(0.06,0.34) and an unstable saddle (0.51,-0.06). As the value increases and $\gamma_m = 0.8$, we have a stable attracting spiral at (0.38 ,0.06) and an unstable saddle at 0.84 and -0.35. Since employment cannot be negative, I focus only on the positive quadrant values. As the value of γ_m increases, the unstable spiral becomes a stable attractor and the unstable saddle remains. The stable spiral remains as long as the value of γ_m is higher than about 0.64 (based on some numerical simulation results). Starting from a region like shown in Figure 3 , in the right top section, wage share decreases and employment share increases(shown by the arrows pointing upward and leftward). The increase in employment puts pressures on wage share as well which then could spiral towards the steady state solution (0.38,0.06) if we are close to the steady state or become unstable. The stable spiral arises in the case where we have complex roots and the $Re(im) < 0$ resulting in a sink. As we decreases the value of γ_m, $Re(im) > 0$ resulting in an unstable spiral. Thus the system bifurcates into an unstable spiral from a stable spiral as the value of γ_m declines. This is a classic case of Hopf bifurcations where the trace of the Jacobian turns from negative(at higher values of γ_m to positive (p 249. Strogatz 2018). For the two systems that are graphed below the Jacobian at the respective steady states is given by :

For $\omega_m^* = 0.38$

$$\begin{bmatrix} -0.3 & -0.8 \\ 0.18 & 0.2 \end{bmatrix}$$

For $\omega_m^* = 0.06$

$$\begin{bmatrix} -0.3 & -0.8 \\ 0.18 & 0.2 \end{bmatrix}$$

Tr(J) < 0 for the first Jacobian while it is greater than zero for the second Jacobian. Moreover, the eigen values cross from the negative to the positive real axis. The eigen value calculations are attached in the Appendix. Note that the Jacobian is computed at the steady state values of ω_m for the different values of γ_m

The figures 4 and 5 finally plot the solutions for the system with two different

FIGURE 3. γ_m = 0.8 *y var-employment x var - labor share*

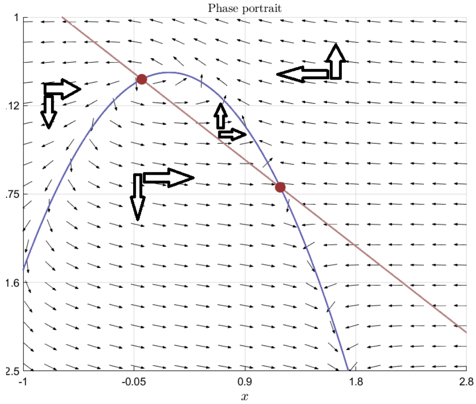
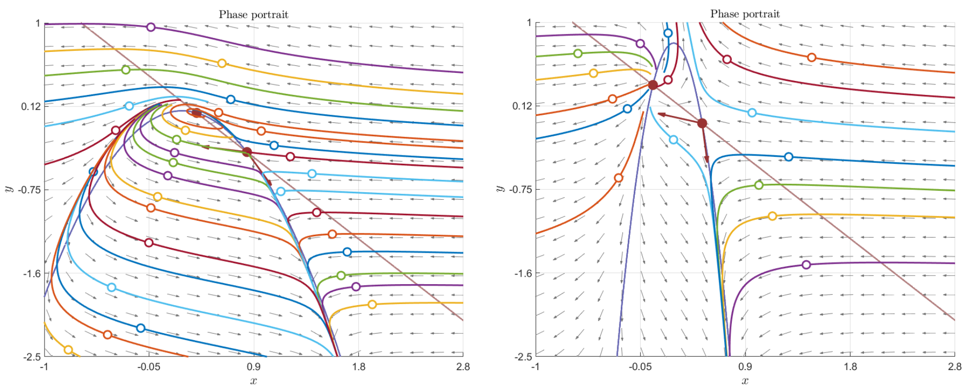


FIGURE 4. γ_m = 0.8 *(left panel) :* γ_m = 0.08 *(right panel)*



values of ω_m (or the sensitivity of labor share growth of manufacturing on service sector employment. As can be seen from the left panel of the figure, there is a stable spiral very close to the fixed point. However, as we grow further away, trajectories could wander off possibly to a distant attractor. The fixed point (0.38 ,0.06) when γ_m = 0.8 is a stable spiral attractor. The system can start from a high labor share high employment regime and spiral to a low labor share and low employment regime, if close to the stable spiral.

IV. Inferences and concluding remarks

In this paper, I attempt to construct a model of labor share and employment share of two sectors.The objective was to bring together the concepts of income distribution(wage share changes) and structural change(change in employment). In the steady state, the model has a stable attracting spiral. Changing the parameter which affects the relationship between labor share and employment changes the stability of the fixed points. Understanding the limits under which such a stable state exists is useful for policy-making implications such as minimum wage policies. Given the simple setup of the model, there are many issues with regards to omission of relevant variables such as growth of capital accumulation (or investment) and an endogenous productivity growth term.

With the broad changes in the US economy, the classical Goodwin model has to an extent broken down(Setterfield 2021) since employment is no longer strictly coupled with labor share. Some of the results of this exercise show that a stability between labor share and employment can only be attained within a close neighbourhood of the fixed point. More complicated non-linear models can be constructed. However, this would require the inclusion of other variables and more reason to understand the applications of non linear dynamics in economics. While these models, might be calibrated with real time data, econometric specification of non-linear models in economics is still a growing area of research and I have not attempted to do that here.Structural change in a 2 sector model can be developed with more realistic assumptions regarding the production of goods and services. Further work in this area could include the link between sectoral prices, output and productivity which will shed more light on the relationship between sectoral changes.

Appendix

Equilibrium stability analysis:

FIGURE 5. $\gamma_m = 0.8$

```
J ~=  |-0.263      -0.8|
      |  0.18       0.2|
with eigenvalues:
 lambda1 = -0.03-0.30i
 lambda2 = -0.03+0.30i
and corresponding eigenvectors
 v1 = | 0.904+0i    |
      |-0.262-0.339i|
 v2 = | 0.904-0i    |
      |-0.262+0.339i|
```

FIGURE 6. $\gamma_m = 0.08$

```
J ~=  |0.374      -0.08|
      | 0.18       0.2|
with eigenvalues:
 lambda1 = 0.29-0 Error messages will be displaye
 lambda2 = 0.29+0.
and corresponding eigenvectors
 v1 = |0.403+0.381i|
      |0.832+0i    |
 v2 = |0.403-0.381i|
      |0.832-0i    |
```

Other Comparative statics : $dL_s/d\omega_m = -\gamma_s/\alpha_s < 0$ Increase in steady state labor share in manufacturing decreases employment share in services.

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